The Size Premium: Australian Evidence

Business Valuation research paper by Macquarie University

MACQUARIE University

CHARTERED ACCOUNTANTS™
AUSTRALIA + NEW ZEALAND

The Size Premium: Australian Evidence

Macquarie University, Applied Finance Centre

Authors: Roland Winn[a], Tony Carlton[b], Jianlei Han[c] and Terry Pan[c]

Purpose: Research Paper for Chartered Accountants Australia and New Zealand

a Senior Lecturer, Applied Finance Centre, Macquarie University;
b Associate Professor, Applied Finance Centre, Macquarie University,
c Senior Lecturer, Department of Applied Finance, Macquarie University.

The authors express their appreciation to Zukai (Michael) Xu, currently completing the Master of Applied Finance, for invaluable research assistance in completing the multiples analysis in this paper.

This research has been assisted by Chartered Accountants Australia and New Zealand ABN 50 084 642 571

Contents

List of figures

List of tables

1. Executive Summary

Scope & background – Should we apply a small firm premium?

- The scope of this paper is to determine whether smaller firms should have a cost of capital premium.
- Standard finance theory typically assumes our relationship with systematic risks, those risks which we cannot diversify away in a portfolio, should be rewarded. Hence, on first glance, a cost of capital premium or additional return expectation for enterprises of smaller size is not consistent with traditional market pricing models.
- We attempt to reconcile these conflicting views in this paper.

What does history tell us?

- Research on the performance of smaller stocks unequivocally demonstrates these are of higher risk than large stocks when risk is measured as volatility of returns. However, when risk is measured as market beta, the results are not as consistent though approximately indicative of higher market risk as well.
- Research on the total return performance of smaller stocks vs larger ones is somewhat mixed. Early U.S. studies demonstrated that smaller stocks tended to outperform but a number of subsequent studies have demonstrated these results may be time dependent.
- Much of the literature indicates that where small firms are observed to exhibit higher returns, it is often coincident with other factors – some of which may be indicators or superior investment strategies and some of which are indicators of risk premiums such as illiquidity or default.

In Practice: What does the market tell us?

- The practice of some valuation practitioners to use firm size as a catch-all for a number of risks is consistent with the finding that it may proxy for a range of factors noted above. Whilst it is preferable to identify and account for each one separately, it may not always be practical to do so.
- Where a premium is applied in Australia, it is typically in the 1.5% to 5% range depending on firm size. By way of comparison, the premiums in the US are 1.0% to 3.6% but with much larger average sized businesses. Note, these are broad generalisations rather than driven from any return-model.
- Our analysis of market multiples indicates that there are differences between the average P/E multiples of small versus larger firms, and the magnitude of the discount in multiples is consistent with market practice. However, when firm specific factors are controlled for the evidence is much weaker.

Conclusions and Recommendations

- We conclude that smaller firms have historically exhibited higher return performance than larger ones. However, it should not be concluded that this is indicative of a replicable investment strategy but rather, indicative of the risks and difficulties of investing in smaller firms.
- Hence we consider that it may be appropriate to apply a cost of capital premium to smaller companies.
- A small firm cost of capital premium is still consistent with the standard CAPM to the extent it is a reward for some risk factor (such as illiquidity) which would otherwise be specified in the numerator of a discounted cash flow (DCF) valuation but is encapsulated as a "catch-all" in the discount rate. This is mathematically equivalent.

2. Background and Motivations

This paper sets out to identify whether a return premium for smaller stocks is justified in a valuation context and, if so, provide guidelines on how the premium can be determined. We note that standard finance theory, premised on the capital asset pricing model (CAPM), makes no allowance for risks other than the market factor. Hence a "size premium" is not consistent with our broadly accepted models for market behaviour and general teaching practices in corporate finance. Where covered, a size premium is often called an anomaly.

However, many market participants employ an "x-factor" to their cost of capital for various reasons including for businesses of smaller size. Other reasons may include liquidity and transactions costs, political or sovereign risk, governance risks, other tail event risks, or accounting for estimation error. These reasons are not mutually exclusive and likely not independent. Some of these reasons relate to actual risks (the premise of our CAPM model) and others relate to market frictions (which are best dealt with explicitly in cashflow forecasts but may sometimes be handled as a short cut in the cost of capital).

This remainder of this paper considers the issues above in the following manner:

- We briefly overview the underpinnings of the discounted cashflow (DCF) valuation methodology and the cost of capital;
- A review of prior research in Australia and internationally on the "size anomaly" is undertaken;
- Empirical analysis of Australian stock returns by different size groupings and control groups is conducted; and
- Premiums implied by market participants are examined.

We conclude by providing some general guidelines that valuation practitioners may employ when considering businesses of smaller size.

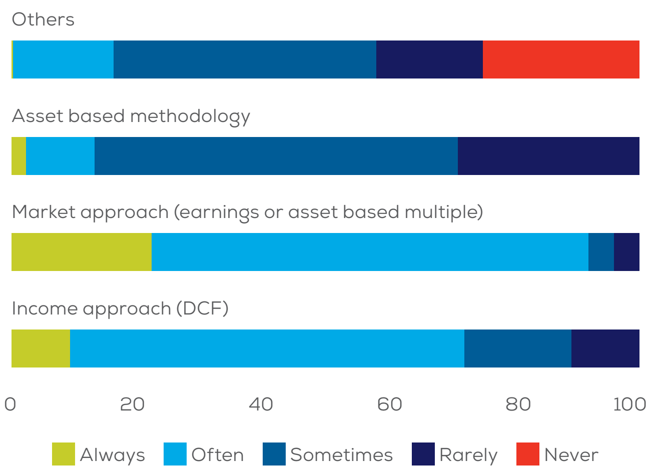
2.1 Motivations and Underpinnings of Cost of Capital

We acknowledge that there are several ways business valuation may be undertaken (see Figure 1) including both multiples analysis and DCF. Whilst the market-based multiples approach is quite popular amongst practitioners, it is readily demonstrated that a multiples type valuation is analogous to a simple, perpetuity DCF (See Supplements Figure 6). Hence, the underpinnings of the cost of capital are important for both DCF and multiples based valuations.

The heuristics employed by market participants in using multiplies also implies a particular cost of capital in addition to implied growth, return on capital and payout ratios. In a DCF framework, these assumptions are set out explicitly and are more than a theoretical construct – they often have quite material consequences. For example, in the case of *Railroad Development Corporation v Republic of Guatemala (ICSID Case No. ARB/07/23) (RDC v Guatemala)*, the difference in applying a size premium of 6.28% and no premium would have changed compensated losses of US$28.9m by approximately US$30m.[1]

1 The tribunal determined a cost of capital of 17.36% in this case with a size premium of 6.28% (see also Rogers (2016)).

Figure 1: Survey of Valuation Approaches Used



Source: KPMG (2017)

Whilst many, but not all, market participants apply a size premium, our standard (CAPM) framework for modelling cost of capital does not provide explicit consideration for size. The most common expression for cost of capital being:

Expected Return =
Risk free (R_f) + β × (Market Risk Premium) + ε [1]

Where

- **β** Captures the systematic risk of any security or business versus the broader market; and
- **ε** is a residual term for other (non-systematic) risks that should not be compensated in an open capital market. The expected, average value should be zero.

The premise of the model above is that most investors can hold a portfolio whereby non-systematic or idiosyncratic risks of any business can be diversified away. [2] Proponents of the use of a size premium would argue that the above model is mis-specified and the modification below is required.

Expected Return =
Risk free (R_f) + β_1 × (Market Risk Premium)
+ β_2 × Size + ε [2]

A key argument is that smaller (single-location) businesses have less ability to withstand economic shocks and require higher returns. However, imagine for example three engineering business – one operating only in Sydney, another in Perth and one that had a national footprint. A West Australian mining boom may improve the operation of the Perth business at the expense of the Sydney one which faced higher labour costs to retain staff, and it could be argued these risks were reasonably balanced for the national firm.

Whilst the Sydney firm suffered, the marginal investor could also hold interests in the Perth business which would have offset the lower NSW margins. At some point, these factors would likely reverse. To the extent the impacts of the mining boom flowed to the broader economy, all three businesses would proportionally share these benefits and costs.

Cashflow Risk vs. the Cost of Capital

The example above provides a simple demonstration of the concept of why, under standard portfolio theory, non-systematic risks (the local conditions in Perth and Sydney) would not be rewarded on average and why only the relationship with systematic risks (the broader, national, market conditions) influence the cost of capital. In this example, the temporary, tighter, margins in Sydney and better conditions in Perth should be modelled explicitly in the derivation of expected cashflows. This introduces two very important concepts related to where to account for risk (table below).

Figure 2: Types of Risk

Type of Risk	How to deal with risk
Firm specific (non-systematic) risk	To the extent there is risk specific to a smaller firm (e.g. the smaller, local state-based firm above), these are scenarios that should be modelled explicitly in expected cashflows.
Market-wide (systematic) risk	To the extent there is a risk that impacts all firms (i.e. cannot be diversified away even in a portfolio of firms), all firms would require a *systematic* compensation relative to how they (in the case above, engineering firms) behave relative to the economy.

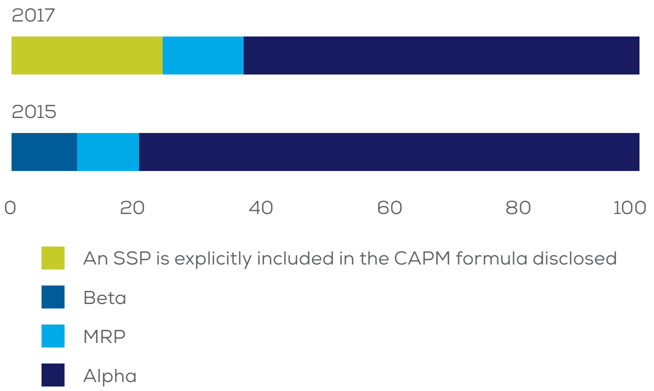
2.2 Survey Evidence

As we have noted, a size premium is not consistent with traditional teaching around cost of capital and DCF. However, as shown below, just under a quarter of market practitioners in Australia employ it as an adjustment to their standard CAPM formula. We note the sample size is limited but it does indicate some discrepancy in valuation methods employed. When practitioners do employ a size premium, it is in the range of 1% to 5% for firms below $250m and $50m equity value respectively. The sharp increase in the micro-cap area is consistent with results in academic literature.

2 The Capital Asset Pricing Model (CAPM), most often attributed to Sharp (1964) and Lintner (1965), was built around the foundations of Markowitz's work on portfolio theory and the benefits of diversification. It is argued risk which may be diversified in a portfolio should not be systematically rewarded i.e. in their cost of capital.

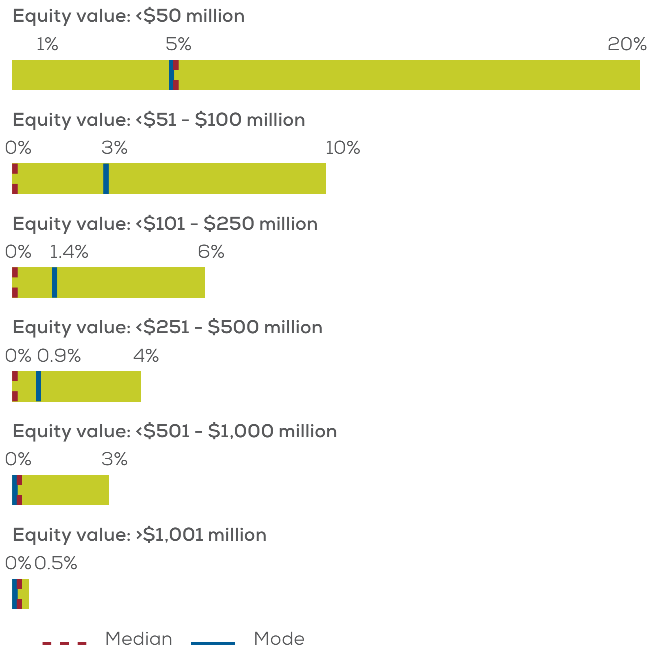
Figure 3: What do valuation practitioners do to adjust for company size?

How often is a size premium applied



Source: KPMG (2017)

What premium is added to the discount rate for size?



2.3 Prior Research

A cost of capital or excess return premium for size has been relatively contentious for many years. It has been one of the most heavily researched topics in finance and we cannot do all the prior papers justice other than to say the topic has been relatively controversial.

One of the most comprehensive compilations of US stock returns is provided by Ibbotson, Duff and Phelps (2016). This is an annually updated reference of US stock returns dating back, in some series, to 1926. This reference, whilst not a peer reviewed journal, is used by many valuation advisors and hence is worth noting given its prevalence and value as a summary data series.[3]

Ibbotson, Duff and Phelps (2016) readily demonstrate the anomaly via what is essentially a taxonomy of "raw returns" over very long periods, parsed by different factors including growth and value investing, liquidity, as well as size. Ibbotson, Duff and Phelps (2016) show smaller firms generate greater returns than larger ones *on average* (table below) over very long periods of time.

Figure 4: U.S. Evidence of Average Annual Returns by Size Groups (1926 – 2015) [4]

Size Group	Threshold (USD)	Premium
Micro-Cap	$448m	3.6%
Low-Cap	$2.08b	1.7%
Mid-Cap	$9.6b	1.0%

However, Ibbotson, Duff and Phelps (2016) note three very important points:

1. Since 1926, large firms beat small firms in almost half the years.
2. Small firms have experienced far greater variability in returns.[5]
3. Smaller firms tend to have more market risk (market beta greater than one) than large firms.

3 Note, Duff & Phelps are a global valuation advisor.

4 Micro-caps, low-caps and mid-caps and defined respectively as deciles 9-10, 6-9 and 3-5 in their analysis. These corresponded to market capitalisations as at 30 September 2015 of up to US$448m, US$2.08b and US$9.6b.

5 This second point is critical as our standard theory suggests raw volatility of returns is not what drives cost of capital but the standardized covariance of the returns with other securities.

Whilst their report is extensive, it is worth noting that the return series examined do not attempt to completely control for all risk factors, nor do they attempt to model or statistically analyse the significance of the results. Our confidence in recommending a premium is reduced by the likelihood the large variability is masking some as yet unidentified factors which influence the return premiums. Hence it is important to examine firm size whilst controlling for other factors which impact valuations and returns. Furthermore, the drivers of a size premium should be pervasive across markets and hence out-of-sample confirmation from different exchanges is required. In addition, we need to demonstrate it is enduring (i.e. not specific to just a particular time period).

Banz (1981) was one of the earliest studies to demonstrate that smaller firms had higher risk-adjusted returns. Fama & French (1993) and Fama & French (2012) further demonstrated that returns to a size based factor were significant when added to a standard market model in addition to the value and momentum factors.[6] Key points to note from Fama & French (2012) are that global models fare poorly (i.e. local conditions matter more when determining expected returns), and the size premium is not consistent across geographies and where present, is not statistically significant. This difference in results from may be attributed to differences in sample periods. In order to do a global, multi-factor, analysis, the sample period had to be limited to 1989 – 2011 whereas earlier data went as far back as 1962.

Latter studies have found conflicting results. Horowitz, Loughran, and Savin (2000) could find no evidence of the size effect using three different methodologies during the period 1980-1996 across the NYSE, NASDAQ and Amex. Cochrane (2005) states

> ***"Many of the anomalous risk premia seem to be declining over time. The small-firm effect completely disappeared in 1980; you can date this as the publication of the first small-firm effect papers or the founding of small-firm mutual funds that made diversified portfolios of small stocks available to average investors."***

Similarly, Ang (2014) states

> ***"The size effect—that small stocks outperform large stocks—was brought to investors' attention by Banz in 1981 and reached its peak after that...Since the mid-1980s, however, there has been no size premium after adjusting for market risk."***

Atanasov and Nitschka (2017) examine global data in addition to the US, Europe and Asia. Atanasov and Nitschka (2017) note some key results: there is a broadly pervasive value effect, there is some evidence of a size effect but that it turns negative for extreme growth stocks. A further nuance is that the book-to-market value factor is rewarded in small stocks but not big stocks and that this small-value premium may be associated with macroeconomic risks.

Cakici and Tan (2014) examine the size, value and momentum factors in 23 developed international stock markets. The don't find size premia in any of the countries examines but do find that find that excess returns from the value and momentum factors decrease with market size. Cakici, Tan and Yan (2016) likewise cannot find evidence of a size factor in emerging markets between 1990 and 2013 except in China.

De Moor and Sercu (2013) argue many earlier studies had filtered out the smallest of stocks which drives the size premium – as indicated by Banz (1981). De Moor and Sercu (2013) demonstrate a post-1980s size effect when employing an international dataset even after controlling for various risk factors. They suggest dividend yield may yet be a key driver of the size effect. However, any analysis may be confounded by the time-varying nature of dividends.

It has been posited that the size effect disappeared due to the publication of academic research[7]. However, this presages a view that it was some form of arbitrage rather than a risk factor or a proxy for a risk factor. The evidence on this appears indeterminate. Closer to home, the results are also somewhat mixed.

6 We will hereafter refer to this type of analysis as a three-, four-, five-, or multi-factor models to mean a CAPM (market factor) model with the addition of various other possible drivers of return explanations including size.

7 Schwert (2003).

Figure 5: Summary of Australian Research on Size Premium

Reference	Start	End	Description	Exists (Y/N)
Gaunt (2004)	1991	2000	Tests three factor model demonstrated size effect exists only in the very smallest cohort. Found raw return premiums of approximately 2% per month between smallest and largest cohort.	Y
Gharghori,Chan and Faff (2007)	1996	2004	Run a three factor model demonstrating small-minus-big portfolio outperformed even after controlling for default probability.	Y
Gharghori, Hamzah and Veeraraghavan (2010)	1991	2006	Find a small cap premium, particularly where they are value stocks which stay in same size cohorts. Average annual size premium of 8.5% (small-minus-big portfolio).	Y
Brailsford , Gaunt and O'Brien(2011)	1982	2006	Conclude there is an insignificant difference in returns between large and small portfolios after controlling for the book – to-market value factor. Find that small-minus-big portfolios actually underperformed by – 0.22% per month.	N
Dou, Gallagher and Schneider (2012)	1992	2010	Conclude micro-cap stocks drive many of the factor anomalies. However, find that the performance is actually negative in small-minus-big type portfolios.	N
Zhong, Limkriangkrai, Gray (2014)	1990	2012	Run a multi-factor model and observe a size premium but find it is highly seasonal in January and July – hence potentially driven by tax effects.	Y
Gaunt (2015)	1974	2013	Concludes prior evidence of the size premium due to highly illiquid, low price stocks which are not realistically investible. When excluding these, the premium disappears.	N
Durand, Limkriangkrai and Chai (2016)	1992	2010	Ran a four and five factor model and did not find significant explanation for the cross-section of Australian returns.	N
Chiah, Chai,Zhong and Li (2016)	1982	2013	Run a five-factor model and do not find significant size effect. However, also note have dropped many of the small firms from the sample in order to conduct statistical analysis.	N

A key finding of many of the results is that the size effect is non-linear – meaning it is biggest in the micro-cap stocks. Some researchers view these stocks as essentially uninvestable for the average portfolio manager, due to their low liquidity and size, and subsequently exclude these from the analysis. Whilst we agree with this notion, we do not agree that we should simply exclude these from our analysis. By excluding the very stocks which demonstrate the premium, we cannot conclude that it does not exist.

Whilst returns to micro-cap stocks may not be appropriate indications of expected return for a fund manager, the data may still be informative in assessing the cost of capital. That is, the fact that returns are so high, in a part of the market that is so difficult to invest in, suggests they may be indicative of the higher hurdle required as compensation for some risk factor such as illiquidity – which is normally far more difficult to proxy.[8] Interestingly, Graham and Harvey (2015) undertake a survey of US chief financial officers (CFOs) and could not find significant differences in the internal WACC estimates and hurdle rates amongst big and small firms. However, as these were internal cost of capital estimates, an external liquidity or uncertainty related risk factor cannot be ruled out – a CFO is unlikely to impute a small firm premium on their own, non-traded, projects.

8 Illiquidity has many definitions depending on the view of the user. In some measures, it is spread or market impact but it may also be as readily defined as the discount to transact an *entire* firm within a defined period. Form this viewpoint, larger enterprises will be more illiquid than small ones. However, this latter definition may be more pertinent at the small-cap end of the market which is comparable to private equity or venture capital type enterprises.

Summary

It should be noted that some of the conclusions from Banz (1981) are still relevant today. Where small firm outperformance is observed:

- It is typically non-linear. Excess returns are most pronounced in the smallest firms.
- The outperformance of small firms is not stable over time.
- A portfolio of small firms typically has much larger residual (i.e. unexplained) risk than a portfolio of large ones when compared to a value-weighted index.[9]
- "We do not even know whether the factor is size itself or whether size is just a proxy for one or more true but unknown factors correlated with size".

The results from the many studies that have followed are, at best, mixed. Our conclusions from prior literature are that:

- There is some evidence that smaller firms exhibit greater returns than larger ones but this result is not pervasive across all geographies or time periods.
- It is readily demonstrated that smaller firms are riskier than larger ones based on the variability of stock prices. Whilst average returns may be higher, the incidence of small firms outperforming large ones may not be large.
- Some of this observed outperformance is coincident with other return factors such as value, profitability and price momentum or risk factors such as credit risk or illiquidity.

These results lead us to be wary of recommending a size premium in the cost of capital without attempting to control for additional risk factors. We will attempt to shed further light on this in an Australian context by reviewing the performance of stocks of different sizes whilst controlling for different valuation and risk factors.

We introduce in our research a control for credit risk as measured by Distance-to-Default (DTD).[10] DTD provides a forward looking estimate of a company's likelihood of default based on option pricing rather than financial variables (such as with an Altman Z-score). Given the expected volatility of a given security is unlikely to change as much as financial variables, it may provide a more consistent, and forward-looking, measure of risk. Gharghori et al used a similar test in estimating a probability of default following Merton (1974). Gharghori et al used returns from mimicking portfolios of high default vs low default firms.

Given the number of now confounding papers in the last decade, we consider it worth revisiting with a more standardised DTD measure likely to be used by market practitioners. In addition, we attempt to include the micro-cap end of the market in our analysis whilst controlling for growth and profitability. In addition to returns analysis, we introduce use of trading multiples to determine whether there is a market-implied premium for small stocks.

The next section describes our data and research design.

9 This is not entirely unexpected as larger firms make up a greater portion of the index. However, it does highlight that small firms behave differently.

10 The DTD concept has its origins in the work of Merton (1974) and Vasicek (1977) where equity is viewed as an option on a business. This credit risk model has been adopted by Moody's amongst others. DTD takes a firm's debt level, expected growth in value and expected volatility of value to estimate likelihood of default.

3. Aim and Data

3.1 Aim

Most of the literature implicitly or explicitly approaches the topic from the perspective of a share market investment manager and whether small firms exhibit some exploitable superior return characteristic. Our motivations are to test whether small firms exhibit systematically different characteristics to large firms such that they warrant a different cost of capital, *all else equal*. To the extent there are observable differences, our objective is to determine how much of that is due directly to the size of a business and how much due to other factors. To the extent small firms demonstrate superior returns, we do not discount these if they are unexploitable, we consider they may reflect compensation for some cost or risk associated with firm size. Hence they are not true excess returns but may help inform our valuation method.

Our research design has two parts explained in detail in the next section.

1. We first conduct returns analysis as per prior research whilst incorporating micro-cap stocks. This analysis is needed to confirm whether small firms continue to outperform large ones and helps us undertake statistical controls for various factors other than the market risk premium and firm size.
2. The second part of our research is based on traded multiples. We examine whether, regardless of share price performance, small firms trade at different capitalisation rates (multiples) to large firms. From this, infer whether there is a market-implied discount factor for smaller stocks.

3.2 Data & Parameters

The data on stock market variables and company financial items are obtained from Datastream and Worldscope. The sample period is from January 1987 to December 2017. Our universe of stocks is larger than that of the standard market indices such as the ASX200 or All Ordinaries Index. We start with every stock listed on the ASX for which we have relevant data.

We construct the monthly return by using the percentage change in Datastream's month-end total return index RI, which counts for both capital gain and dividends for each stock in the sample. We measure firm size as the market value in the preceding December. Following Watanabe et al. (2013), asset growth rate (AG) is measured as the percentage change in total assets from the end of fiscal year *t-2* to the end of fiscal year *t-1*. We measure return-on-assets (ROA) as the ratio between net income and total asset in the end of fiscal year *t-1*.

We source DTD data for Australian firms from the CRI database, the Credit Research Initiative of the National University of Singapore, available at: http://rmicri.org (accessed 05-07-2018). For our multiples analysis, we obtain data from Capital IQ.

4. Returns Anlaysis Research: Design and Results

4.1 Research Design

Our returns analysis is conducted by dividing the stocks into quintiles based on market capitalisation in 2016. We maintained these dollar value cut-offs in our groupings throughout the years. This was chosen to provide market practitioners a dollar number by which they could differentiate their valuations. We start by simple segmentation of total returns and excess returns ("alphas") under different size buckets versus financial risk (DTD), profitability (ROA) and growth. We then conduct multivariate analysis of the form:

$$R_{i,t} - R_{f,l,t} =$$
$$a + \beta_1(R_m - R_{f,l,t}) + \beta_2\, DTD + \beta_3\, ROA + \beta_4\, Growth$$
$$+ \beta_5\, Size\ \varepsilon_{i,t} \quad [3]$$

This allows us to identify whether smaller firms earn a premium after controlling for risk and valuation factors. However, it is our expectation that the association, if any, will be non-linear and concentrated at the very small end. Hence we respecify (1) into the form:

$$R_{i,} - R_{f,l,t\ t} =$$
$$a + \beta_1(R_m - R_{f,l,t}) + \beta_2\, DTD + \beta_3\, ROA + \beta_4\, Growth$$
$$+ D_1 Micro + D_2 Small + \varepsilon_{i,t} \quad [4]$$

Where Micro is represented by our first size bucket (quintile 1) and Small our second size bucket (quintile 2) and Di represents dummy variables of 0 or 1.[11] Estimating a coefficient for a broad size bucket is likely to be of greater use to market practitioners.

4.2 Summary Statistics from Returns Analysis

Our summary data is broadly described in Table 1. As expected, outside the standard market indices, there is a very large tail of small caps with the median market capitalisation of $31m vs an average of $729m.

In breaking the data into size buckets, we can see that the following trends occur as companies get smaller:

- Market risk, beta, is materially higher for the Micro-caps (the smallest quintile);
- Returns versus the biggest quintile increase monotonically;
- Credit risk (measured as probability of default) versus the biggest quintile increase monotonically;
- Average profitability versus the biggest quintile is negative and decreases monotonically; and
- Asset growth is lowest for the micro-caps.

The last point is of particular interest. The higher returns of the micro-caps is not due to normal growth but rather, their option-like qualities. This has implications for research design which are worth exploring in future analysis.

11 Note, for completeness, we also analysed (2) with dummy variables for each of the four groupings either excluding quintile 1 or quintile 5. They do not change our conclusions.

Table 1: Firm Characteristics by Size

Size Bucket*	Market Capitalisation up to: ($m)	Monthly Total Returns	St. Dev	Beta	Excess Returns	DTD**	ROA	Asset Growth
1 Micro	6.15	8.81%	14.6%	1.23	6.02%	1.97	-0.62	-0.03
2 Small	17.04	5.24%	12.7%	1.06	3.80%	2.56	-0.35	0.10
3 Mid	50.02	3.92%	7.7%	1.08	2.55%	3.18	-0.32	0.17
4 Large	253.14	2.47%	5.4%	1.01	1.38%	4.09	-0.13	0.18
5 Biggest		1.35%	4.5%	1.04	0.00%	5.37	0.38	0.17

* We recognise the terminology will not align with common market practice for dividing stocks into four size buckets but have used them for pragmatic convenience to differentiate our five size buckets. In normal market ASX vernacular, our micro and small would be considered collectively as micro, our mid cap would be termed small cap, our large cap would be termed mid cap and there is clearly no "biggest" grouping.

** A lower DTD indicates higher risk.

Table 2 presents results for Realised Returns sorted by the size groups. The absolute magnitude of monthly returns reported below is quite large. These numbers clearly do not reflect replicable portfolios. In conducting the analysis, we have not taken into account transactions costs, liquidity or taxes amongst other frictions. However, the magnitude of the returns is in line with other studies of the Australian market. For example, Dou et al (2013) report a small-minus big premium of 2.8% per month for their micro cohort of $22m average market capitalisation stocks. This is against our mid-minus-biggest of 2.6% and small-minus-biggest of 3.9%.

Table 2: Returns Analysis Summary Statistics[12]

Statistic	N	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Re	246,278	0.015	0.211	-0.080	0.000	0.068
R_{market}	246,278	0.014	0.040	-0.010	0.018	0.040
Beta	177,590	1.020	1.060	0.416	0.936	1.581
MV	246,278	729.426	4,850.038	8.530	31.330	172.340
DTD	171,700	3.592	2.445	1.911	3.145	4.692
ROA	205,855	-0.200	9.398	-0.247	-0.033	0.053
Growth	196,448	0.122	0.664	-0.104	0.047	0.252

12 Means are equal weighted.

We sorted the firms into four groups based on similar DTD scores (i.e. not an ordinal rank). These results are presented in Tables 3 and 4. Firms of similar credit risk are in each of the columns below. Higher returns to smaller firms was observed even when they were of similar financial risk. Firms of higher financial risk tended to have higher returns but not in all sub-groups. In future analysis, it may be worth incorporating multiple credit metrics.

Table 3: Returns by Size and Financial Risk

Size Bucket	Market Capitalisation up to: ($m)	Q1 Highest Risk	Q2	Q3	Q4 Lowest Risk
1 Micro	6.15	8.29%	9.50%	7.29%	5.36%
2 Small	17.04	5.62%	5.10%	5.02%	5.28%
3 Mid	50.02	5.32%	4.57%	3.75%	2.65%
4 Large	253.14	4.02%	2.84%	2.46%	2.09%
5 Biggest		1.78%	1.75%	1.33%	1.20%

*Financial Risk measured by standardised Distance-to-Default scores

Table 4: "Alpha" by Size and Financial Risk

Size Bucket	Market Capitalisation up to: ($m)	Q1 Highest Risk	Q2	Q3	Q4 Lowest Risk
1 Micro	6.15	6.90%	7.75%	5.48%	3.96%
2 Small	17.04	4.30%	3.51%	3.39%	4.05%
3 Mid	50.02	3.69%	3.03%	2.27%	1.18%
4 Large	253.14	2.49%	1.28%	0.98%	0.77%
5 Biggest		0.16%	0.21%	-0.16%	-0.17%

Tables 5 and 6 show results of sorting firms based on Return on Assets (ROA). Sorting the firms by profitability did not appear to lead to material differentiation in returns. There appears, at best, a weak association between ROA and returns. Unsurprisingly then, smaller firms continued to exhibit higher returns after controlling for profitability.

Table 5: Returns by Size and ROA

Size Bucket	Market Capitalisation up to: ($m)	Q1 Lowest ROA	Q2	Q3	Q4 Highest ROA
1 Micro	6.15	8.75%	9.40%	7.26%	8.60%
2 Small	17.04	4.76%	5.48%	3.69%	7.58%
3 Mid	50.02	4.00%	4.73%	3.02%	4.30%
4 Large	253.14	2.00%	2.55%	1.99%	2.63%
5 Biggest		1.62%	1.41%	1.66%	1.56%

Table 6: "Alpha" by Size and ROA

Size Bucket	Market Capitalisation up to: ($m)	Q1 Lowest ROA	Q2	Q3	Q4 Highest ROA
1 Micro	6.15	7.01%	7.74%	6.58%	7.06%
2 Small	17.04	3.10%	3.81%	2.50%	6.24%
3 Mid	50.02	2.49%	2.41%	1.68%	3.06%
4 Large	253.14	0.35%	0.65%	0.63%	1.40%
5 Biggest		-0.71%	-0.50%	-0.20%	0.29%

Tables 7 and 8 present results of sorting groups by size and growth. We observe a tendency for faster growing firms to exhibit higher returns. Whilst not quite monotonic across all groups, the relationship appears strong and appears to be higher as firms get smaller. This is more evident when looking at the alphas. Controlling for growth does moderately reduce the size effect in returns but there remains a strong, observable size premium.

Table 7: Returns by Size and Growth

Size Bucket	Market Capitalisation up to: ($m)	Q1 Lowest Growth	Q2	Q3	Q4 Highest Growth
1 Micro	6.15	7.73%	8.66%	8.43%	10.35%
2 Small	17.04	4.87%	4.10%	6.24%	6.04%
3 Mid	50.02	3.00%	3.76%	4.20%	5.56%
4 Large	253.14	2.48%	1.72%	1.98%	3.31%
5 Biggest		1.34%	1.19%	1.29%	1.62%

Table 8: "Alpha" by Size and Growth

Size Bucket	Market Capitalisation up to: ($m)	Q1 Lowest ROA	Q2	Q3	Q4 Highest ROA
1 Micro	6.15	5.94%	6.91%	6.76%	8.96%
2 Small	17.04	3.26%	2.95%	5.19%	3.94%
3 Mid	50.02	1.36%	2.56%	2.70%	3.53%
4 Large	253.14	0.96%	0.47%	0.66%	1.70%
5 Biggest		0.07%	-0.29%	-0.15%	0.11%

4.3 Regression Model

We conducted four regression models based on expression (1) to control for the simultaneous effects of size, profitability, financial risk and growth. Table 9 demonstrates that market returns are very significant drivers of firm level returns. However, we continue to observe a strong, negative, association between firm size and returns – larger firms make lower returns, smaller firms greater returns.

We have used the log of market capitalisation (MV) in our analysis. We can therefore compare the percentage change in size against returns. However, care must be taken as the larger the percentage change, the less reliable will be the model coefficient but in brutally simple terms, regression (1) indicates, for example, a 10% change in market value is associated with a 0.05% decline in monthly expected returns.

The addition of DTD in regression (2) modestly weakens the relationship between size and returns. Hence at least some of the total returns from smaller firms is due to financial risk. Given we have only used one measure, effectively probability of bankruptcy, it would be worth exploring more holistic measures of financial risk.

We then introduce profitability which weakens the significance of our size variable but has not real change on our coefficient. Whilst profitability appears significant in (3), when we introduce growth as well into regression (4) we find it is no longer significant – consistent with the tables above. We do find that growth is also a driver of returns but, interestingly, size continues to be significant.

Table 9: Multi-factor Regressions of Monthly Returns

	Dependent variable: R_e-R_f			
	(1)	(2)	(3)	(4)
R_m-R_f	1.035***	1.108***	1.086***	1.101***
	t = 99.139	t = 89.798	t = 80.284	t = 80.332
log(MV)	-0.005***	-0.004***	-0.004***	-0.004***
	t = – 25.290	t = – 15.137	t = – 13.506	t = – 14.149
DTD		-0.001***	-0.001***	-0.001***
		t = – 4.356	t = – 3.817	t = – 3.848
ROA			0.0004**	0.0001
			t = 2.163	t = 0.766
Growth				0.007***
				t = 7.217
Constant	0.019***	0.018***	0.018***	0.019***
	t = 22.067	t = 16.764	t = 15.236	t = 15.233
Observations	246,278	167,552	137,631	133,117
R^2	0.041	0.049	0.047	0.049
Adjusted R^2	0.041	0.049	0.047	0.049
Note:	*p<0.1; **p<0.05; ***p<0.01			

Our analysis indicates that smaller firms earn higher returns even after accounting for financial risk, growth and profitability. However, prior studies in addition to our summary statistics indicates this result is likely highly concentrated in the micro cap universe. Whilst this segment is outside of standard indices, and virtually uninvestable for the average fund manager, a return premium may still be of interest to those required to value businesses of smaller size. We estimate expression (2) with dummy variables for micro cap and small cap stocks in place of market size. Results are shown in Table 10.

Table 10: Regression with Micro and Small Cap vs Mid Cap and Above

	Dependent variable: R_e-R_f			
	(1)	(2)	(3)	(4)
R_m-R_f	1.030***	1.102***	1.081***	1.095***
	t = 99.105	t = 89.759	t = 80.335	t = 80.387
Size dummy1 (micro)	0.037***	0.037***	0.037***	0.040***
	t = 34.660	t = 24.776	t = 22.837	t = 23.919
Size dummy2 (small)	0.008***	0.008***	0.008***	0.010***
	t = 7.272	t = 6.367	t = 5.831	t = 6.740
DTD		-0.001***	-0.001***	-0.001***
		t = – 4.739	t = – 4.127	t = – 4.227
ROA			0.0004**	0.0001
			t = 2.355	t = 0.575
Growth				0.009***
				t = 9.433
Constant	-0.008***	-0.004***	-0.004***	-0.005***
	t = – 14.569	t = – 3.531	t = – 3.183	t = – 4.400
Observations	246,278	167,552	137,631	133,117
R^2	0.042	0.050	0.049	0.051
Adjusted R^2	0.042	0.050	0.049	0.051
Note:	*p<0.1; **p<0.05; ***p<0.01			

By choosing to use dummy variables for just micro and small cap, we are comparing each to all other stocks collectively in size buckets 3 to 5 (mid cap to largest).[13] In practical terms, the mid-cap to largest buckets in our study represent the investable universe for average market participants on the ASX. Both micro and small cap stocks are significantly different to this investable universe. Further, the coefficient on our micro dummy was approximately four times that of our small dummy. The absolute of the coefficients, which suggests 3.7% and 0.8% monthly premium are again not achievable for a fund manager.

As noted, the lack of investability and the perceived disbelief in the quantum of returns has led many researchers to exclude these stocks from their analysis. Other reasons are a lack of data to properly conduct a multi-factor model. However, we have chosen to err on the side of including as much of the known universe as possible. Despite the limitations of the data (e.g. possibility of stale prices and likely very large transactions costs), we consider there to be a very strong non-linear relationship between returns and size primarily driven by the very smallest of stocks.

These stocks rarely trade and are more venture capital like in nature. Before we discount the observed returns as being unrealistic, if we examine the returns of early stage venture capital managers, we find their returns are also of significant magnitude. The 20, 25 and 30 year *realised* returns of early stage U.S. venture capital managers are reported as 70.2%, 41.7% and 25.1% respectively per annum as reported by Cambridge Associates (2018) which maintains one of the most comprehensive databases of private equity funds available. Ideally we would like to compare to Australian performance but matching data of equal quality is currently unavailable. It would be worth exploring in future analysis if performance of Australian venture capital managers has similar characteristics to our micro and possibly small buckets or attempt to replicate our expanded listed market analysis with US stock market and venture capital data.[14]

Our regression indicates that there is a micro and small firm size effect but we would not be confident in using the coefficients in a cost of capital as: (1) the quantum of implied returns will essentially discount to an exceptionally short payback period; and (2) we need to account for the option-like payoffs.

In order to provide some guidance for hurdle rate estimates, we will look to see if can observe differences in average price multiples for firms of different size. This may then yield an implied capitalisation rate difference which can be imputed into cost of capital size premium.

13 For robustness, we also ran the regression with dummy variables for all size cohorts except micro cap to test the reverse hypothesis that it is a large or biggest cap effect instead. We found the coefficients on cohorts 3 through 5 significantly different to micro caps but of equal magnitude. Hence, it supports are model design to use dummy variables on micro and small only. Further, the fact size cohort 3 which would normally be considered small cap in many studies did not have a different coefficient to the largest stocks – hence consistent with a lack of a "small firm anomaly".

14 Cambridge Associates maintains a historical record of over 2,000 fund managers and 7,300 funds in addition to capturing the gross performance of some 80,000 firm level investments in the private markets space (venture, private, growth, buyout and subordinate capital).

5. Market Implied Premiums: Research Design and Results

5.1 Size premium and multiples: motivation

Research into size premia has generally focussed on realised returns, consistent with the analysis in this paper. However, an alternative way to test for the presence of a size premium is to examine market multiples. The relationship between a Discounted Cash Flow valuation and the Price Earnings multiple presented in Supplement 7.1, shows that the Price Earnings multiple incorporates an embedded, or implied, discount rate (or Required Return). If the Required Return includes a size premium then, for a given dividend forecast, the multiple will be lower. We can therefore test for the presence of a size *premium* (in Required Returns) by the testing whether smaller firms have a *discount* in their valuation multiples[15].

Table 11 shows the relationship between different size premia incorporated into the Required Return, and the impact on the resultant multiple, for a given Base Discount Rate (no size premium) and a given payout ratio. The table shows that for growth rates in the 3% to 4% range, a 3% size premium should result in a 30 % to 33% reduction in multiples[16]. This conclusion only holds if the growth rate is constant across the groups. If smaller size firms have systematically higher growth rates then there may be no apparent discount, even if there is a size premium embedded in the Required Return.

Table 11: Relationship between Growth Rate, Payout Ratio and Size Premium

MULTIPLES FROM ALTERNATIVE COMBINATIONS OF SIZE PREMIUM AND GROWTH RATE									
		Base K (no size premium): 10%						Payout Ratio: 50%	
Growth				**Size premium embedded in Required Return**					
Rate	**0%**	**1%**	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**
0%	5.0	4.5	4.2	3.8	3.6	3.3	3.1	2.9	2.8
1%	5.6	5.0	4.5	4.2	3.8	3.6	3.3	3.1	2.9
2%	6.3	5.6	5.0	4.5	4.2	3.8	3.6	3.3	3.1
3%	7.1	6.3	5.6	5.0	4.5	4.2	3.8	3.6	3.3
4%	8.3	7.1	6.3	5.6	5.0	4.5	4.2	3.8	3.6
5%	10.0	8.3	7.1	6.3	5.6	5.0	4.5	4.2	3.8
6%	12.5	10.0	8.3	7.1	6.3	5.6	5.0	4.5	4.2
7%	16.7	12.5	10.0	8.3	7.1	6.3	5.6	5.0	4.5
8%	25.0	16.7	12.5	10.0	8.3	7.1	6.3	5.6	5.0
9%	50.0	25.0	16.7	12.5	10.0	8.3	7.1	6.3	5.6
10%		50.0	25.0	16.7	12.5	10.0	8.3	7.1	6.3

15 In the balance of this paper we will continue to refer to the presence of a size premium embedded in the Required Return, to be consistent with the terminology used thus far.
16 These results apply only to the growing perpetuity model. The impact of a given size premium would be different for assets with other growth profiles and durations.

To test for this effect, Cornell and Gokhale (2018) analysed the Price/Earnings multiples ("P/E Multiple") for companies listed on the NYSE/AMEX and NASDAQ exchanges over the period 2007 to 2014. Companies were split into ten size deciles, consistent with the Ibbotson classification. They analysed P/E multiples on a year by year basis, as well as pooling the observations over the 2007 to 2014 period. The model tested by Cornell and Gokhale is shown in Equation 5:

$$P/E_t = \alpha + \beta_1 NIGrowth_t + \beta_2 D_j + \beta_3 Years_y + \varepsilon_{i,t} \quad [5]$$

P/Et multiples were based on Last Twelve Month's Earnings. Growth in Net Income ($NIGrowth_t$) was measured by the three year Compound Growth Rate in Net Income, with higher growth expected to be correlated with higher P/E multiples. D_j represents Dummy variables for the different size groups. $Years_y$ is included to control for the effects of individual years.

Table 12 presents the key results for the pooled sample, which included observations for all years. The coefficient for Growth in Net Income was positive, as expected. Dummies for size groups 2 through 10 were included, with decile 10 representing the smallest group. This allows us to measure the impact of size relative to the reference group of the largest decile of firms (decile 1)[17]. A size effect should see the dummy variables with negative values, and the coefficient should get more negative as we move from decile 1(large) to decile 10(small). However, the results show that, after allowing for differences in Growth in Net Income, each of the coefficients was positive, and statistically significant. The co-efficient for decile 10 is lower than the other deciles suggesting that the smallest group has a lower multiple than deciles 2 through 9, but it is still positive relative to the largest size group (decile 10).

Table 12: Summary Results for Cornell & Gokhale (2018) Pooled Regression

P/E Multiple	Intercept	NI Growth	Deciles 2 – 9	Decile 10 (small)
Coefficient	20.90***	28.08***	3.09 to 4.67	1.87
t values	22.97***	60.96***	2.95 to 4.95***	2**

These results contradict the hypothesis that size premia are embedded in valuations. The results were robust to alternative specifications of the model. P/E multiples based on forward earnings were also tested but the results were similar to the LTM basis. Additional analyses included Beta as an independent variable, but was found to have no impact. The analysis was completed for individual years, and demonstrated the same broad results.

There are two issues with the Cornell and Gokhale method. First, by using P/E multiples, the research design automatically excludes companies with negative incomes. There is no data provided as to whether there is a systematic trend in companies with smaller market capitalisations to have negative incomes. From our sample of Australian data, we observe that a higher proportion of smaller firms have negative incomes. So excluding firms with negative incomes may exclude the firms where the size effect may be most prevalent. Second, the only asset specific variable included is Income Growth rate. There are other valuation models that have good explanatory power which should allow us to better identify whether size has an incremental impact on value. Rhodes-Knopf, Vishwanathan and Robertson (2005) ("RKRV") developed a regression model based on the Residual Income model. This model is explained in Supplement 7.3. One advantage of this model is that it allows firms with negative incomes to be included in the sample. This model has been used to successfully measure valuation impacts of mergers and acquisitions. However, the model can be used whenever it is necessary to control for firm specific variables. These applications have been used to directly estimate the dollar amounts of Enterprise Value and Market Capitalisation, but they can be applied to explain multiples as well, with appropriate modification. We use this model as a complement to the Cornell and Gokhale (2018) methodology.

5.2 Research Design

5.2.1 Do Australian P/E multiples reflect a size premium?

We carried out a similar test using Australian data. Financial data was extracted from Capital IQ for firms listed on the ASX with a financial reporting date over calendar years 2010 to 2017. Companies were then split into five size groups, using the same cutoff levels applied for the returns analysis earlier in the paper. Table 13 presents summary financial metrics for each of the size groups.

In breaking the firms into size groups, we can make the following observations about the financial characteristics of the different size groups:

- Revenue and Invested Capital increase in line with the size groups based on Market Capitalisation;
- The largest group, which has a cutoff Market Capitalisation of $253 million, is clearly of a different scale in terms of key accounting metrics;
- On measures of income, EBITDA and Net Income, the four smallest groups are close to zero or commonly negative;
- Standard Deviations are material, reflecting a wide range of results in each group.

17 This is the same procedure used earlier in this paper to test for a relationship between realised returns and size. However, Cornell and Gokhale (2018) went from large (decile 1) to small (decile 10).

Table 13: Financial Characteristics of firms (2010-2017) [$m]

Size Bucket*	N	Revenue	St. Dev.	EBITDA	St. Dev.	Net Income	St. Dev.	Total Common Equity	St. Dev.
1 Micro	2,368	3.05	16.07	-0.93	2.71	-2.55	8.86	5.74	10.57
2 Small	2,430	8.88	31.91	-1.05	4.66	-3.62	12.29	11.66	16.94
3 Mid	2,609	24.75	128.71	-1.00	10.55	-5.82	29.79	25.59	39.66
4 Large	2,646	95.65	248.54	8.03	28.53	-7.08	53.95	84.79	122.51
5 Biggest	2,808	2,295.70	7,036.67	478.16	2,147.41	246.74	1,215.56	2,429.39	7,376.31

*Size bucket cutoffs are the same as those for the returns analysis earlier in this paper

The initial analysis uses the P/E multiple calculated in the Capital IQ database. These are based on normalised LTM Net Income, and only calculated for firms with positive Incomes. Results are reported in Table 14. We can make the following observations:

- There is a clear trend towards increasing Price Earnings multiples as the size groups get bigger, consistent with the presence of a size premium;
- There is a large jump from Size Group 2 to Size Group 3, suggesting two broad size cohorts, a conclusion we drew in the analysis of Realised Returns;
- There is a wide variation within each group, particularly the small size groups. The standard deviation of the small size firms is larger, in absolute and relative terms, for the four smallest groups relative to the group of large firms;
- There is significant sample loss in the small size groups due to need to include only observations with a positive P/E multiple. The number of observations is materially reduced in the smaller groups due to the prevalence of firms reporting negative incomes. The final column in Table 14 shows the percentage of observations that are lost due to the need to calculate Price Earnings multiple using positive incomes.

Table 14: Price Earnings Multiples by Size Group (2010-2017)

Size Bucket	Mean	St. Dev	Median	Pct(25)	Pct(75)	N	Percent of original sample with Negative Incomes
1 Micro	20.11	35.14	8.43	1.49	22.95	223	89.6
2 Small	21.82	32.06	12.41	6.69	23.79	381	81.9
3 Mid	28.79	41.51	15.26	8.69	31.72	628	76.0
4 Large	27.51	36.26	16.10	10.80	27.86	1,262	53.9
5 Biggest	25.64	27.54	20.17	12.96	29.79	2,395	19.7

A.Mean, is the Arithmetic Mean *St. Dev.* is the Standard Deviation. *Pct(25)* and *Pct(75)* represent the 25th and 75th percentile respectively. *N* is the number of observations. Companies with negative incomes were those which reported a negative Net Income in Capital IQ database.

In addition to the traditional Arithmetic Mean (simple average) we calculated the Geometric Mean and Harmonic Mean, as they are less influenced by outliers. Table 15 shows the results. These alternative measures all confirm the initial result, that the P/E multiple increases as we move to firms included in the larger size buckets.

If we take the average of the two smallest size groups, and compare to the average of the three larger size groups we can estimate an overall multiple discount applied to small firms. The bottom row of Table 15 shows these estimates range between 23% and 58%, an average of 42%.

Table 15: Price Earnings Multiples with Alternative Summary Measures

Size Bucket	Mean	Median	G. Mean	H. Mean	N
1 Micro	20.11	8.43	12.47	5.52	223
2 Small	21.82	12.41	14.34	7.58	381
3 Mid	28.79	15.26	18.94	12.77	628
4 Large	27.51	16.10	19.47	14.62	1,262
5 Biggest	25.64	20.17	23.14	19.51	2,395
Implied Discount of Groups [1] & [2] relative to others	23%	39%	48%	58%	

G. Mean and *H. Mean* represent the Geometric and Harmonic Mean respectively.

The results for the Standard Deviation and Interquartile Range (25% and 75% percentiles) suggest there is significant overlap between the groups. To test whether the differences between groups are significant, we completed a pairwise comparison of means for each group. Table 16 presents the results. There are three conclusions to draw from this analysis:

- there is a significant difference between the two smallest size groups;
- the two small size groups are significantly different to the other three, larger, size groups;
- between the three larger size groups, the differences are not significant.

These results are consistent with the earlier results in the paper, which identified differences between the smaller groups and the cohort of larger companies.

Table 16: Comparison of Price Earnings Multiples by Size Group

Tests for differences between means for each size bucket					
	1 Micro	2 Small	3 Mid	4 Large	5 Biggest
1 Micro	NA	-0.5935	-3.0142***	-2.8855**	-2.2862**
2 Small		NA	-2.9878**	-2.9461***	-2.2045**
3 Mid			NA	0.6542	1.7962
4 Large				NA	1.6042
5 Biggest					NA

Values in each cell are t statistics testing for differences in means of each size group. Results with either a ** or *** are significantly different at the 5% or 10% level of significance, respectively. For example, the top right hand corner shows that there is a significant difference between the P/E multiple for the Micro group stocks and the Biggest group of stocks. t-test is calculated as always [Smaller group] – [Larger group], so a negative sign implies the smaller group has a lower multiple, consistent with the presence of a size premium.

5.2.2 Analysis using other multiples

In order to address the loss of sample due to negative incomes, we carried out similar tests using the Enterprise Value to Revenue (EV to Revenue) and the Market Capitalisation to Total Common Equity (Market to Book) multiples. Both these measures allow the inclusion of companies with negative incomes. The EV to Revenue multiple was extracted directly from CapIQ, while the Market to Book multiple was calculated using Market Capitalisation and Total Common Equity extracted from CapIQ.

Tables 17 and 18 present the results of this analysis. For both multiples the column headed N shows the larger sample size, and the more even spread of observations over the groups. For the EV to Revenue multiple, neither the Arithmetic Mean or Harmonic Mean show any pattern, the median shows an increase as size groups get bigger, and the Geometric Mean shows a decrease.

Table 17: EV to Revenue Multiple by Size Groups (2010-2017)

Size Bucket	EV to Revenue					
	AM	St. Dev.	Median	GM	HM	N
1 Micro	26.20	53.53	0.86	7.98	0.88	1,990
2 Small	27.89	54.76	4.99	6.89	0.82	2,078
3 Mid	29.38	58.84	6.42	6.16	0.97	2,172
4 Large	18.95	47.38	6.60	3.59	0.96	2,335
5 Biggest	7.56	22.16	9.91	3.00	0.96	2,706

For the Market to Book multiple, the Arithmetic Mean shows no pattern however all the other measures show an increase over the size groups, consistent with the presence of a size premium in market multiples. Overall, these results provide some evidence of a size premium being incorporated into market multiples but it is not as consistent as for the Price Earnings multiple.

Table 18: Market to Book Multiple by Size Groups (2010-2017)

Size Bucket	Market to Book Ratio					
	AM	St. Dev.	Median	GM	HM	N
1 Micro	1.57	9.90	0.65	0.79	0.40	2,226
2 Small	84.36	3,801.92	1.25	1.54	0.83	2,362
3 Mid	7.97	627.28	1.73	2.19	1.07	2,546
4 Large	8.25	100.97	1.98	2.48	1.26	2,618
5 Biggest	6.68	80.71	2.67	2.67	1.79	2,723

To test the significance of these differences we carried out the same pairwise comparison test we did for the Price Earnings multiple. There was no evidence of any differences for the Market to Book multiple. Results for the EV to Revenue are presented in Table 19. These results suggest there are significance differences between each for the size groups for the EV to Revenue multiple but the different signs show that the pattern is not consistent. For example, a negative sign implies that the multiple of the smaller size group is smaller then the comparison group. The results in Table 19 suggest that the smallest size groups have an EV to Revenue multiple lower than the next larger groups, 2 and 3, consistent with the size premium hypothesis. However, the larger groups have significantly lower EV to Revenue multiples, a result at odds with the size premium hypothesis.

Table 19: Comparison of EV to Revenue Multiple across Size Groups (2010-2017)

Tests for differences between means for each size bucket					
	1 Micro	2 Small	3 Mid	4 Large	5 Biggest
1 Micro	NA	-1.0029	-1.8320*	4.6669***	14.6372***
2 Small		NA	-0.8538	5.7635***	15.9529***
3 Mid			NA	6.5214***	16.3759***
4 Large				NA	10.6599***
5 Biggest					NA

Values in each cell are t statistics testing for differences in means of each size group. Results with either a *,** or *** are significantly different at the 1%, 5% or10% level of significance, respectively.

t-test is calculated as always [Smaller group] – [Larger group], so a negative sign implies the small group has a lower multiple, consistent with the presence of a size.

5.3 Regression Model

The results so far are univariate tests, and so do not control for any stock specific variables. The analysis in Section 4.2 .1 demonstrated it is important to control for differences in Growth Rates in order to identify a true size premium. To control for stock specific factors, we estimated a regression model similar to Cornell and Gokhale (2018), by including an estimate of Growth Rate in Net Income. We used the 2 year Compound Growth rate in Net Income as provided in Capital IQ data base. Beta was also included as an explanatory variable, using the Capital IQ 5 year Beta . Dummy variables for the two smallest groups were included to test for the size effect; the coefficients will show the impact of size relative to a reference group of the three larger size groups. Allowance was also made for industry membership, using the eleven Capital IQ Primary Sectors, and years (2010 to 2017). The following model was tested:

$$P/E_t = \alpha + \beta_1 NIGrowth_t + \beta_2 D_j + \beta_3 Years_y + \beta_4 Industry_i + \varepsilon_{i,t} \quad [6]$$

As noted earlier, the RKRV model is potentially a richer valuation model than one which just includes growth rates. The full model is explained in the Supplements 7.3. The model essentially includes combinations of Income and / or Total Common Equity, depending on the multiple being used. It also allows for the inclusion of forms with negative incomes. Tests were also carried out using the RKVR model. For the P/E multiple the following model was tested:

$$P/E_t = \alpha + \beta_1 NIGrowth_t + \beta_2 \ln(Total\ Common\ Equity_t) + \beta_4 D_j + \beta_5 Years_y + \beta_6 Industry_i + \varepsilon_{i,t} \quad [7]$$

Where $\ln(Total\ Common\ Equity_t)$ is a logarithm of each firm's Total Common Equity (book value of shareholder's equity). This model was also tested by using the Ln(P/E) as the dependent variable.

Results

Table 20 presents results for the different models for the Price Earnings Multiples. The results for the size coefficients are mixed. The coefficient for Size Group 1 is insignificant in all the models, however the coefficient for Size Group 2 is significant in Model [3][18]. The value of the coefficient makes economic sense. In Model [2], the value of the coefficient for Size Group 2 of – 4.64 suggests that members of Size Group 2 suffer a 4.64 absolute reduction in P/E multiple relative to the reference group of the larger firms in Size Groups 3, 4 and 5. This is in line with the results in Table 14. The coefficient value in Model [3] suggest membership of Size Group 2 attracts an approximately 25% discount in multiple relative to the same reference group. We conclude there is some evidence for a size premium, but it is not as evident as looking at the simple averages as we did earlier.

18 It is also significant in other specifications of the model not reported here.

Table 20: Regression Results for Price Earnings Multiple (2010-2017)

	Dependent variable		
	[1]	[2]	[3]
	Price Earnings	Price Earnings	Ln(Price Earnings)
Net Income CAGR	-0.0079	-0.0086	-0.0002
	t=-2.71***	t=-2.93	t=-2.17**
Beta	-1.6948	-0.8021	-0.0379
	t=-1.58	t=-0.72	-1.29
Ln(Total Common Equity)		-0.8754	0.0104
		t=-2.82***	t=1.54
Size Dummy 1	3.5639	0.7395	-0.1913
	t=0.43	t=-0.09	t=-1.10
Size Dummy 2	-2.7914	-4.6406	-0.2482
	t=-1.05	t=-1.59	t=4.13***
Constant	16.74	20.41	2.65
	t=11.65***	t=11.00	t=48.09***
Industry Effects Included	Yes	Yes	Yes
Year Effects Included	Yes	Yes	Yes
Number of Observations	3,028	3,000	2,999
Adjusted R^2	0.046***	0.051***	0.104***

Items identified with *,** and *** are significance levels at 10%, 5% and 1% respectively

In terms of other results, the only consistent conclusion is that the coefficient for the 2 year compound growth rate in Net Income is negative, which is counter to expectations, and the coefficient for Beta is insignificant in all the models.

We completed the same analysis for the EV to Revenue and Market to Book multiples. The results are presented in Table 21. For the EV to Revenue Multiple (Models [4] and [5]) coefficients for the size variables show that the smaller Size Groups have a higher EV to Revenue Multiple. This is consistent with the univariate results presented earlier, but at odds with the size premium hypothesis. For the Market to Book Multiple (Model [7]), both size coefficients have negative values in line with the size premium hypothesis. If the values for these coefficients are interpreted loosely as the percent impact on the Market to Book Multiple, the values are broadly consistent with the results in Table 18.

Table 21: Regression Results for EV to Revenue & Market to Book Multiples (2010-2017)

	Dependent variable			
	[4]	[5]	[6]	[7]
	EV to Revenue	Ln(EV to Revenue)	Market to Book	Ln(Market to Book)
Net Income CAGR	0.0003	0.00002	-0.0103	0.0002
	t=3.21***	t=3.21***	t=-1.02	T=1.78**
Beta	-0.1322	-0.0946	-19.57	-0.1610
	t=-0.07	t=-1.99**	t=-1.06	t=-3.71***
Ln(Absolute Income)	1.5513	-0.0456	-50.5242	-0.1044
	t=-1.02	t=-3.26***	t=-1.28	t=-9.92***
Negative Income Indicator	0.1629	0.1629	-0.0103	-0.1564
	t=3.23***	t=3.23**	t=-1.08	t=-2.86***
Ln(Invested Capital)	-4.8472	-0.0874		
	T=-6.14***	T=-3.26		
Size Dummy 1	3.2520	0.5814	-327.4000	-0.6158
	t=0.71	t=3.286***	t=-1.22	t=-4.46***
Size Dummy 2	1.0920	0.3439	238.7722	-0.3710
	t=0.38	t=2.39**	t=0.87	t=-4.17***
Constant	21.55	1.39	171.42	1.60
	t=7.42***	t=12.06***	t=1.38	t=30.76***
Industry Effects Included	Yes	Yes	Yes	Yes
Year Effects Included	Yes	Yes	Yes	Yes
Number of Observations	3,794	2,718	4,288	3,314
Adjusted R^2	0.145***	0.222***	0.036	0.168**

Items identified with *,** and *** are significance levels at 10%, 5% and 1% respectively

In terms of the other variables, the coefficients for Growth Rate in Net Income and Beta are in line with expectations. The results for the Negative Income Indicator are mixed. For the market to Book Multiples (Models [6] and [7]) the coefficient is negative, as expected. They are the opposite sign for the EV to Revenue Multiple (Models [4] and [5]).

Estimating the implied size premium

It is possible to convert a discount in the P/E multiple into an implied Size Premium. Using the simplest valuation model, the perpetuity Dividend Valuation model presented in Supplement 7.1, it is possible to back out an implied premium, making assumptions about Payout ratio and Growth rate. Table 11 presents results for one set of scenarios. To demonstrate, using the 40% average discount calculated in Table 15, the analysis in Table 11 suggests a 40% discount equates to a size premium of between 2% and 4%, for a given combination of payout ratio and growth rate. If one assumed that smaller firms have lower payout ratios and higher growth rates, then the embedded size premium would need to be larger to explain a 40% difference between multiples of small versus large firms.

6. Conclusions

We find significant evidence of smaller firms earning higher returns than larger firms. Like prior research, the results are non-linear. The returns are concentrated in the very small end of the stock universe. We find the results to hold even after controlling for redit risk, growth and profitability.

We do not consider this to reflect a superior investment strategy as such a portfolio cannot be practically replicated. However, we consider that it may be indicative of the need to apply a higher cost of capital premium for smaller firms, particularly micro-cap businesses.

Our results are different to recent research but this may due to our attempt to include as many of the micro-cap stocks as possible. We do this as, from a valuation perspective, many market practitioners are faced with the dilemma of how to properly value small, essentially private, businesses. Second, the fact that it is very difficult to replicate such a portfolio indicates the difficulty in trading such businesses requires a higher hurdle rate.

We observe, unsurprisingly, that smaller firms are very thinly traded and that their security prices are more volatile. The higher return premium to small firms may therefore be attributable to liquidity in addition to other unknown or difficult to measure factors such as information quality, credit rationing. Even the seasonality effect, not tested here, is consistent with concentrated selling pressure effects.

We also tested whether size premia might be embedded in market multiples. Based on simple averages of different size portfolios, we find that smaller firms have lower P/E multiples, a pattern consistent with the Returns analysis. However using regression analysis, which controls for other factors, the evidence is less compelling. There is some evidence of size premia, but results are sensitive to the actual model used. Further testing is required before a stronger conclusion van be drawn.

We consider that it would be prudent for valuation practitioners to attempt to identify and estimate as many of the explicit costs and risks as possible rather than blindly apply a simple shortcut. However, for pragmatic reasons, a cost of capital size premium, acting as a catch-all for a number of other effects, has some merits.

A number of issues are worth considering in future analysis. In particular, better estimating liquidity costs under different conditions (e.g. market traded c versus transactional), differentiating between different economic regimes, and incorporating a more holistic view of credit risk would be worthwhile.

7. Supplements

7.1 A price multiple is really just a simple DCF

A price multiple is analogous to a perpetuity DCF valuation with the difference being whether growth, cost of capital and earnings are considered explicitly or implicitly. This is demonstrated in the steps in the figure below.

Figure 6: Reconcile Market Multiples to Single Period DCF

Perpetuity Valuation of Equity Based on Dividends	P_0 = Dividend$_1$ / k – g
Substitute EPS_1 * Payout Ratio for Dividend$_1$	P_0 = EPS_1 * Payout Ratio / k – g
Divide Across by EPS gives Forecast PE	P_0/EPS_1 = Payout Ratio / k – g

Where

P_0	is the current share price;
Dividend$_1$	is the one-year forward expected dividend;
Payout Ratio	is the expected dividend payout ratio;
k	is the cost of equity;
g	is the perpetuity growth rate; and
EPS_i	is the Earnings Per Share at time ***i***.

7.2 Additional Tables: Regression of Large vs Small

Table 22: Regression Results for Realised Returns with Dummy Variable for all Size Groups

	Dependent variable: Re			
	(1)	(2)	(3)	(4)
Mkr	1.031***	1.104***	1.082***	1.096***
	t = 98.865	t = 89.582	t = 80.064	t = 80.118
Size dummy2	-0.030***	-0.028***	-0.028***	-0.030***
	t = − 21.981	t = − 16.296	t = − 15.085	t = − 15.409
Size dummy3	-0.037***	-0.036***	-0.036***	-0.039***
	t = − 27.395	t = − 21.291	t = − 19.337	t = − 20.173
Size dummy4	-0.038***	-0.037***	-0.038***	-0.041***
	t = − 28.992	t = − 21.882	t = − 20.335	t = − 21.461
Size dummy5	-0.037***	-0.035***	-0.035***	-0.038***
	t = − 27.748	t = − 19.139	t = − 17.415	t = − 18.516
DTD		-0.001***	-0.001***	-0.001***
		t = − 4.661	t = − 4.052	t = − 4.082
ROA			0.0004**	0.0001
			t = 2.317	t = 0.582
Growth				0.009***
				t = 9.298
Constant	0.029***	0.032***	0.033***	0.034***
	t = 29.998	t = 23.401	t = 21.569	t = 21.905
Observations	246,278	167,552	137,631	133,117
R^2	0.043	0.051	0.049	0.052
Adjusted R^2	0.043	0.051	0.049	0.052
Note:	*p<0.1; **p<0.05; ***p<0.01			

7.3 Valuation using a regression approach

Rhodes-Kropf, Robinson and Viswanathan (2005) start with the Residual Income valuation expression. Making some assumptions about growth rates of assets and income, and assuming mean reversion of returns towards the required return, it is possible to specify the following regression model:

$$EV_t = \alpha_{0t} + \alpha_{1t} IC_t + \alpha_{2t}\, Earnings_t \quad [8]$$

EV represents the Enterprise Value, IC represents the Invested Capital and Earnings represents the after tax operating earnings of a firm. The equity valuation counterparts of these variables are Market Capitalisation, Total Common Equity and Net Income.

Rhodes-Kropf, Robinson and Viswanathan (2005) convert to logs to account for skewness in the accounting data. They also allow for the inclusion of negative income firms by using absolute values for income, and an indicator variable for negative income targets. Allowing this expression to be estimated over time, ***t***, and industries, ***j***, gives the following specification of:

$$ln(EV_{jt}) = \alpha_{0,jt} + \alpha_{1,jt} ln(IC_{jt}) + \beta_{2,jt} ln(Earnings)^{+}_{jt} + \alpha_{3,jt} I_{(<0)} ln(Earnings)^{+}_{jt}\, \varepsilon_{jt} \quad [9]$$

$Earnings+_{jt}$ represents the absolute value of earnings, while the fourth term includes an indicator variable equal to 1 for firms with negative income. The coefficients in these regressions will be proportional to discount rates and growth rates.

Rhodes-Kropf, Robinson and Viswanathan (2005) use this model to analyse the value impacts of public company mergers and acquisitions, however the model ahs wider applicability as it allows for the incorporation of firm specific characteristics into a regression based valuation model. Expression [3.16] would be converted to a multiple by dividing EV_t by either one of the right hand side variables.

8. References

Ang, A. (2014), "Asset Management: A Systematic Approach to Factor Investing", Oxford University Press, p.457.

Atanasov, V. and Nitschka, T., (2017), "Firm size, economic risks, and the cross-section of international stock returns", The North American Journal of Economics and Finance, 39, pp.110-126.

Banz, R. W. (1981), "The relationship between return and market value of common stocks", Journal of Financial Economics 9, pp.3-18.

Brailsford, T., Gaunt, C. and O'Brien, M.A. (2012), "Size and book-to-market factors in Australia", Australian Journal of Management, 37(2), pp.261-281.

Cakici, N., Tang, Y. and Yan, A., (2016), "Do the size, value, and momentum factors drive stock returns in emerging markets?", Journal of International Money and Finance, 69, pp.179-204.

Cakici, N. and Tan, S., (2014), "Size, value, and momentum in developed country equity returns: Macroeconomic and liquidity exposures", Journal of International Money and Finance, 44, pp179-209.

Cambridge Associates (2018), "US Venture Capital Index and Selected Benchmark Statistics March31, 2018", Copyright © 2018 Cambridge Associates.

Chiah, M., Chai, D., Zhong, A. and Li, S., (2016), "A Better Model? An empirical investigation of the Fama–French five-factor model in Australia", International Review of Finance, 16(4), pp.595-638.

Cochrane, J. (2005), Asset Pricing: Revised Edition (Princeton University Press), p.452.

Cornell, B. and Gokhale, R. (2018), "Do Valuation Multiples Reflect a Size Effect", Journal of Business Valuation and Economic Loss Analysis, Volume 13, Issue 1, February: pp.1 – 23.

De Moor, L. and Sercu, P. (2013), "The smallest firm effect: An international study", Journal of International Money and Finance, 32, pp.129-155.

Durand, R.B., Limkriangkrai, M. and Chai, D. (2016), "The Australian asset-pricing debate", Accounting & Finance, 56(2), p393-421.

Dou, P.Y., Gallagher, D.R. and Schneider, D.H., (2013), "Dissecting anomalies in the Australian stock market", Australian Journal of Management, 38(2), pp.353-373.

Fama, E.F. and,French, K.R.,(1993), "Common risk factors in the returns on stocks and bonds", Journal of Financial Economics 33, 3–56.

Fama, E. and French, K. (2012), "Size, value, and momentum in international stock returns", Journal of Financial Economics 105, 457-472.

Gaunt, C. (2015), "The illusory Australian small firm premium", JASSA The Finsia Journal of Applied Finance 2, p19-25.

Gharghori, P., Chan, H.W.H., Faff, R.W., (2007), "Are the Fama-French factors proxying default risk?", Australian Journal of Management 32, 223–249.

Gharghori, P., Hamzah, Y. and Veeraraghavan, M., 2010. Migration and its contribution to the size and value premiums: Australian evidence. Journal of International Financial Markets, Institutions and Money, 20(2), pp.177-196.

Graham, J. R. and Harvery, C. R. (2015), "The Equity Risk Premium in 2015", Duke University Annual CFO Survey.

Horowitz, J.L., Loughran, T. and Savin, N.E., 2000. Three analyses of the firm size premium. Journal of Empirical Finance, 7(2), pp.143-153.

Ibbotson, R., Duff and Phelps (2016), "2016 Stocks, Bonds, Bills and Inflation©", Wiley.

KPMG (2017), "For all it's worth: KPMG Valuation Practices Survey 2017", KPMG International.

Lintner, J. (1965), "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets". Review of Economics and Statistics. 47 (1): 13–37.

Merton, R., (1974), "On the pricing of corporate debt: the risk structure of interest rates", Journal of Finance 29, 449–470.

Rogers, D (2016), "The Size Premium is 35: Has It Grown Up?", The European, Middle Easter and African Arbitration Review, p16-20.

Rhodes-Kropf, Matthew; David T. Robinson and S. Viswanathan (2005), "Valuation waves and merger activity: the empirical evidence." *journal of financial economics*, 77(3), pp 561-603.

Schwert, G. W. (2003), "Anomalies and market efficiency" in Constantinides, G.M., Harris, M., Stulz, R.M. (Eds.), Handbook of the Economics of Finance. North Holland, Amsterdam.

Sharpe, W. F. (1964), "Capital asset prices: A theory of market equilibrium under conditions of risk". Journal of Finance. 19 (3): pp425–442.

Vasicek, O. (1977), "An equilibrium characterization of the term structure", Journal of Financial Economics,5, 177–188.

Watanabe, A., Xu, Y., Yao, T., and Yu, T. (2013), "The asset growth effect: Insights from international equity markets", *Journal of Financial Economics*, 108(2): 529-563.

Further enquiries

Australia: 1300 137 322

New Zealand: 0800 4 69422

service@charteredaccountantsanz.com

Follow and tweet us
@Chartered_Accts

Follow us
Chartered Accountants Australia and New Zealand

Follow us
Chartered Accountants Australia and New Zealand

CHARTERED ACCOUNTANTS™
AUSTRALIA + NEW ZEALAND

853-SEP-18